Distributed Simultaneous Coverage and Communication Control by Mobile Sensor Networks

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***Abstract*—The purpose of this paper is to propose a distributed control scheme to maximize area coverage by a mobile robot network while ensuring reliable communication between the members of the team. The information that is generated at the sensors depends on the sensing capabilities of the sensors as well as on the frequency at which events occur in their vicinity, captured by appropriate probability density functions. This information is then routed to a fixed set of access points via a multi-hop network whose links model the probability that information packets are correctly decoded at their intended destinations. The proposed distributed control scheme simultaneously optimizes coverage and routing of information by decoupling coverage and routing control. Specifically, optimization of the communication variables is performed periodically in the dual domain. Then, between communication rounds, the robots move according to the solution of a distributed sequential concave program that handles efficiently the introduced nonlinearities in the mobility space. Our method is illustrated in computer simulations.**

I. Introduction

The area coverage problem has recently received a lot of attention and the related literature is quite extensive. In [1], the authors propose a distributed controller based on Lloyd's algorithm for sensing a convex area. In this work, it is assumed that the sensing performance degrades as the distance from the sensor increases. The case where the robots are equipped with range-limited sensors is discussed in [2]. Coverage optimization for anisotropic sensors is studied in [3], [4], while [5]–[7] discuss coverage of non-convex areas. Common in this literature on area coverage problems is that it typically ignores the requirement that the information collected by the robot sensors needs to be routed to a desired set of destinations. In this paper, we provide a distributed solution to the problem of joint coverage and communication control.

We assume a team of mobile robot sensors responsible for covering a convex area of interest with the additional requirement that the sensory information collected by the robots can be efficiently routed to a desired set of fixed access points (APs). The rate of information generated at every sensor depends on the quality of sensing as a function of the sensing range, as well as on the probability that events occur in the vicinity of that sensor, captured by an appropriate probability density function over the area of interest. On the other hand, routing of this information to the APs is via a dedicated multi-hop network whose links model the probability that information packets are correctly decoded at their intended destinations. Unlike existing methods that focus on preserving graph connectivity [8]–[11], our approach to communication control employs more realistic communication models, motivated by [12], [13]. The proposed distributed control scheme utilizes only information that is locally available at the sensors in order to simultaneously optimize coverage and routing of information to the APs. The key idea is to decouple coverage and routing control and alternate between optimization of the two objectives. In particular, given a spacial configuration of the robots in the area of interest, the communication variables are updated using a distributed subgradient algorithm in the dual domain. Then, the robots move in a direction that optimizes the coverage objective. Robot motion is formulated as a distributed sequential concave program, that allows us to handle nonlinearities in the mobility space that are present due to the coverage objective and the communication constraints. As the robots move, the optimal solution in the communications space drifts, which introduces a possible infeasibility gap in the primal variables. While such an infeasibility gap persists, the affected robots remain stationary until feasible routing variables are determined by the optimization in the communications space.

The problem of simultaneous coverage and communication control is also addressed in [14], although in a centralized setting. A related problem that considers the minimization of the aggregate information delivered directly, in one hop, from the robots to a sink node is addressed in [15]. Multi-hop communication in the context of coverage is considered in [16] and [17]. These latter approaches differ from the one proposed here in that we consider more realistic models of wireless communication that involve routing of information over a network of varying link reliabilities, and we also ensure desired information rates that depend on the frequency with which events occur in the sensors' vicinity.

II. Problem Formulation

Assume a team of N mobile robots responsible for the sensing coverage of a convex and compact area $\mathcal{A} \subset \mathbb{R}^2$ and for the transmission of packets of information to a fixed set of K access points (APs). The positions of all nodes are stacked in the vector $\mathbf{x} = [\mathbf{x}_1^T, \dots, \mathbf{x}_i^T, \dots, \mathbf{x}_{N+K}^T]^T$, where $i \in \{1, ..., N\}$ for the robots and $i \in \{N+1, ..., N+K\}$ for the APs. The motion of the robots is assumed to be governed by the first order differential equation

$$\dot{\mathbf{x}}_i(t) = \mathbf{u}_i(t), \quad i = 1, \dots, N, \tag{1}$$

where $\mathbf{u}_i \in \mathbb{R}^2$ stands for the control input associated with the i-th robot.

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To achieve area coverage, each robot is equipped with an isotropic sensor whose accuracy is captured by a radially decreasing function f that is maximal at the sensor location $\mathbf{x}_i$. In this context, a larger value of f means better accuracy. In particular, we choose

$$f(\mathbf{x}_i, \mathbf{q}) = e^{-\|\mathbf{q}-\mathbf{x}_i\|^2}. \tag{2}$$

Moreover, let $\phi(\mathbf{q}) : \mathcal{A} \to \mathbb{R}_+$ be an integrable density function representing the probability that an event takes place at the point $\mathbf{q} \in \mathcal{A}$. Then, the coverage problem can be formulated as follows:

$$\underset{\mathbf{x},\mathcal{W}}{\text{maximize}} \ \mathcal{H}(\mathbf{x}, \mathcal{W}) = \sum_{i=1}^{N} \int_{\mathcal{W}_i} f(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q}, \tag{3}$$

where $\mathcal{W}_i$ is a subregion of $\mathcal{A}$ assigned to robot i for sensing purposes and $\mathcal{W} = \cup_{i=1}^{N} \mathcal{W}_i$ denotes the collection of these regions generated by the robots positioned at $\mathbf{x}$.

The problem that we address in this paper is the optimization of the objective $\mathcal{H}$ in (3), subject to communication constraints required to ensure desired information flows from the sensor robots to the access points (APs). In particular, let $R(\mathbf{x}_i, \mathbf{x}_j)$ be a link reliability metric denoting the probability that a packet transmitted by the i-th robot is correctly decoded by the j-th node. Assuming that the transmission rate of the terminals' radios is unity and common for all robots, the effective transmission rate from i to j is also equal to $R(\mathbf{x}_i, \mathbf{x}_j)$. Moreover, let $r_i \in [0, 1]$ denote the normalized average rate (information units per unit of time) at which the i-th robot generates information. We assume that this rate depends on both the sensing performance over the ϕ-weighted area $\mathcal{W}_i$ and the probability that an event will occur at each point $\mathbf{q} \in \mathcal{W}_i$ so that

$$r_i(\mathbf{x}_i, \mathcal{W}_i) = \int_{\mathcal{W}_i} f(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q}. \tag{4}$$

Packets generated at the terminal i are transmitted to terminal j according to routing probability T_{ij} representing the probability that the i-th robot selects robot j as a destination for its transmitted packets. Upon generation or arrival from another robot, packets are assumed to be stored in a queue at each robot and they leave this queue provided they are transmitted and correctly decoded by any other node j. Thus, the normalized rate at which packets leave the queue at the i-th node and are conveyed to the j-th node is $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$, since the transmission and the decoding process are two independent events. Packets can be conveyed by the i-th robot to the APs either directly if the probability $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$ for $j \in \{N+1, \ldots, N+K\}$ is reasonably large or through a multi-hop communication path. Then, the average rate at which packets leave the i-th queue is $r_i^{out} = \sum_{j=1}^{N+K} T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$. Similarly, the average rate at which packets arrive at the i-th queue is $r_i^{in} = r_i(\mathbf{x}_i, \mathcal{W}_i) + \sum_{j=1}^{N} T_{ji}R(\mathbf{x}_j, \mathbf{x}_i)$. Note that the APs can only receive information which explains the upper limits in the sums that appear in r_i^{in} and r_i^{out}. A necessary condition to ensure that the queue at node i empties infinitely often with probability one is that $r_i^{in} \leq r_i^{out}$. Therefore, packets are almost surely eventually delivered to the APs as long as

$$c_i(\mathbf{x}_i, \mathcal{W}_i, \mathbf{T}) = r_i^{out} - r_i^{in} \geq 0, \ \forall i \in \{1, \ldots, N\}, \tag{5}$$

where $\mathbf{T} \in \mathbb{R}^{N(N+K)}$ is the stack vector of all routing probabilities T_{ij}.

Note that the maximization of the coverage objective (3) subject to the communication constraints (5) is an optimization problem with respect to the robot positions $\mathbf{x}_i$, the routing probabilities T_{ij}, and the partition of the area in regions $\mathcal{W}_i$. In the absence of the constraints, it is well known that the objective function $\mathcal{H}$ is maximized if the partition $\mathcal{W}_i$ is chosen to be the Voronoi partition [18], denoted by $\mathcal{V}_i$, of the space; see Proposition 2.13 in [19]. On the other hand, in the presence of the constraints, the Voronoi regions are not necessarily feasible and, therefore, the feasible optimal partition for the constrained problem is in general different from the Voronoi partition. However, if we are able to ensure feasibility of the Voronoi partition, then this partition will be optimal for the constrained optimization problem. In our problem, this is possible by appropriately selecting the routing probabilities T_{ij}.

Moreover, note that for a given spatial configuration $\mathbf{x}$, the set of constraints (5) may be satisfied by various routing variables T_{ij}. However, introducing a strictly concave objective function $V_{ij}(T_{ij})$ associated with the variable T_{ij}, we can ensure uniqueness of the solution T_{ij}. Incorporating in the optimization problem (3) objective functions $V_{ij}(T_{ij})$, the routing constraints (5), the probability constraint $\sum_{j=1}^{N+K} T_{ij} \leq 1$, and replacing the partition $\mathcal{W}$ in the coverage objective $\mathcal{H}$ and in the communication constraints by the Voronoi partition $\mathcal{V}$ we obtain the following constrained optimization problem: [1]

$$\begin{aligned} \underset{\mathbf{x},\mathbf{T}}{\text{maximize}} \quad & \mathcal{H}(\mathbf{x}) + \sum_{j=1}^{N} \sum_{j=1}^{N+K} V_{ij}(T_{ij}) \\ \text{subject to} \quad & c_i(\mathbf{x}, \mathbf{T}) \geq 0 \\ & \sum_{j=1}^{N+K} T_{ij} \leq 1, \quad 0 \leq T_{ij} \leq 1, \end{aligned} \tag{6}$$

where the constraints in (6) hold for all robots $i \in \{1, \ldots, N\}$. Note that for a fixed spatial configuration $\mathbf{x}$, the reliabilities $R(\mathbf{x}_i, \mathbf{x}_j)$ are fixed and, therefore, the problem in (6) attains a concave form. For the strictly concave function $V_{ij}(T_{ij})$, we choose $V_{ij} = -w_{ij}T_{ij}^2$ encouraging the distribution of the packets over different links [12].

Assume now that the network is initially deployed so that the constraints (5) are satisfied. Then, in this paper we seek a solution to the following problem:

[1] Comparing to (3) and (5), in (6) we have dropped the dependence of the objective $\mathcal{H}$ and the constraints c_i on the Voronoi partition $\mathcal{V}$. The reason is that, unlike any arbitrary partition $\mathcal{W}$, the Voronoi partition is completely determined by the robot positions $\mathbf{x}$. Note also that for the computation of the rate $r_i(\mathbf{x})$ in the constraint $c_i(\mathbf{x}, \mathbf{T}) \geq 0$ only information acquired by the set of Delaunay neighbors is required.

Problem 1: Determine robot positions $\mathbf{x}_i$ and routes $\{T_{ij}\}_{j=1}^{N+K}$ such that coverage is optimized and reliable communication with the APs is guaranteed, as per the solution of problem (6).

III. Distributed Optimal Communication

A centralized solution of (6) as in [14] can incur large communication cost and delays due to the need of identifying the network topology and communicating it to the robots. Therefore, a distributed solution is preferred, where (6) is solved locally across the group of nodes. For this purpose, assuming a fixed network topology denoted by $\mathbf{x}$ we define the Lagrangian of (6) as[2]

$$\mathcal{L}_{\mathbf{x}}(\boldsymbol{\lambda}, \mathbf{T}) = \sum_{i=1}^{N} \sum_{j=1}^{N+K} V_{ij}(T_{ij}) \tag{7}$$
$$+ \sum_{i=1}^{N} \boldsymbol{\lambda}_i \left[\sum_{j=1}^{N+K} T_{ij} R(\mathbf{x}_i, \mathbf{x}_j) - \sum_{j=1}^{N} T_{ji} R(\mathbf{x}_j, \mathbf{x}_i) - r_i(\mathbf{x}) \right],$$

where $\boldsymbol{\lambda} \in \mathbb{R}^N$ is a column vector of the Lagrange multipliers and $\mathbf{T} \in \mathbb{R}^{N \times N+K}$ is a matrix of routing probabilities. Note that the Lagrangian defined in (7) can be expressed as a sum of local Lagrangians $\mathcal{L}_{\mathbf{x},i}$ through reordering its terms, which depend only on variables $\{T_{ij}\}_{j=1}^{N}$, as in [12], i.e.,

$$\mathcal{L}_{\mathbf{x}}(\boldsymbol{\lambda}, \mathbf{T}) = \sum\nolimits_{i=1}^{N} \mathcal{L}_{\mathbf{x},i}(\boldsymbol{\lambda}, \mathbf{T}).$$

Since the optimization problem (6) is concave for fixed robot positions, we can equivalently work with the dual problem. Following the steps of [12], introduce an index k and the time instants t_k at which the communication variables are updated, and define the following distributed gradient descent algorithm in the dual domain:

Primal Iteration For a given spatial configuration $\mathbf{x}(t_k)$ and Lagrange multipliers $\boldsymbol{\lambda}(t_k)$, compute Lagrangian maximizers $\{T_{\mathbf{x}(t_k),ij}\}_{j=1}^{N+K}$ as:

$$\{T_{\mathbf{x},ij}(t_k)\}_{j=1}^{N+K} = \underset{\sum_{j=1}^{N+K} T_{ij} \leq 1}{\operatorname{argmax}} \; \mathcal{L}_{\mathbf{x}(t_k),i}(\boldsymbol{\lambda}(t_k), \mathbf{T}). \tag{8}$$

Dual Iteration Given the primal variables $\{T_{\mathbf{x},ij}(t_k)\}_{j=1}^{N+K}$ from (8), update the dual variables as:

$$\lambda_i(t_{k+1}) = \mathbb{P}\Bigg[\lambda_i(t_k) - \epsilon\Bigg(\sum_{j=1}^{N+K} T_{ij}(t_k) R(\mathbf{x}_i(t_k), \mathbf{x}_j(t_k))$$
$$- \sum_{j=1}^{N} T_{ji}(t_k) R(\mathbf{x}_j(t_k), \mathbf{x}_i(t_k)) - r_i(\mathbf{x}(t_k))\Bigg)\Bigg]. \tag{9}$$

where $\mathbb{P}$ denotes the projection to the non-negative orthant. Note, that the algorithm (8)-(9) is distributed, since it requires only the Lagrange multipliers λ_j (equation (8)) and the routing variables T_{ji} (equation (9)) from robots for which $R_{ij} \neq 0$. In the next section, we integrate communication control with robot mobility for area coverage maximization.

[2]Since we assume a fixed network topology $\mathbf{x}$, the term $\mathcal{H}(\mathbf{x})$ in the objective function of (6) is a constant; therefore, for the sake of simplicity, in the construction of the Lagrangian, it can be ignored.

IV. Coverage and Routing Control

To jointly optimize coverage and communication we propose a hybrid scheme that decouples the two control objectives and alternates between optimization of the two. Specifically, at each time instant t_k, the routing variables are updated via the distributed algorithm (8)-(9) and during the time intervals (t_k, t_{k+1}) the robots move towards configurations $\mathbf{x}_i(t_{k+1})$ that optimize coverage. Since, the update (8)-(9) ensures feasibility of the primal variables for a static network as $k \to \infty$, for any arbitrary finite k and for a mobile network, the primal variables $\{T_{ij}\}_{j=1}^{N+K}$ are not necessarily feasible. This means that the communication constraint c_i may become negative as the i-th robot moves from $\mathbf{x}_i(t_k)$ to $\mathbf{x}_i(t_{k+1})$. To ensure that this error does not grow large and, therefore, that an acceptable quality of communication is maintained, every robot needs to check feasibility of its local routing variables after every communication update. Robots for which these routing variables are infeasible remain stationary until the iteration (8)-(9) returns feasible routes. When feasible routes are obtained, those robots compute their next position $\mathbf{x}_i(t_{k+1})$ and start moving towards it.

Motion planning is via the solution of local sequential concave programs that allow to handle the nonlinear coupling of the robots positions in the optimization problem (6). In particular, assuming that all other robots are fixed at positions $\mathbf{x}_j(t_k)$ for $j \neq i$, every robot i solves the following problem:

$$\begin{aligned} \underset{\mathbf{x}_i}{\text{maximize}} \quad & \tilde{\mathcal{H}}(\mathbf{x}_i, \{\mathbf{x}_j(t_k)\}_{j \neq i}) \\ \text{subject to} \quad & \tilde{c}_i(\mathbf{x}_i, \{\mathbf{x}_j(t_k)\}_{j \neq i}, \mathbf{T}) \geq 0, \\ & \|\mathbf{x}_i - \mathbf{x}_i(t_k)\| \leq \sigma, \end{aligned} \tag{10}$$

for the next position $\mathbf{x}_i(t_{k+1})$, where $\{\mathbf{x}_j(t_k)\}_{j \neq i}$ is the collection of positions at time t_k for nodes $j \neq i$. In (10), $\tilde{\mathcal{H}}$ and $\tilde{c}_i$ are a concave and linear approximation of $\mathcal{H}$ and c_i, respectively. Specifically,

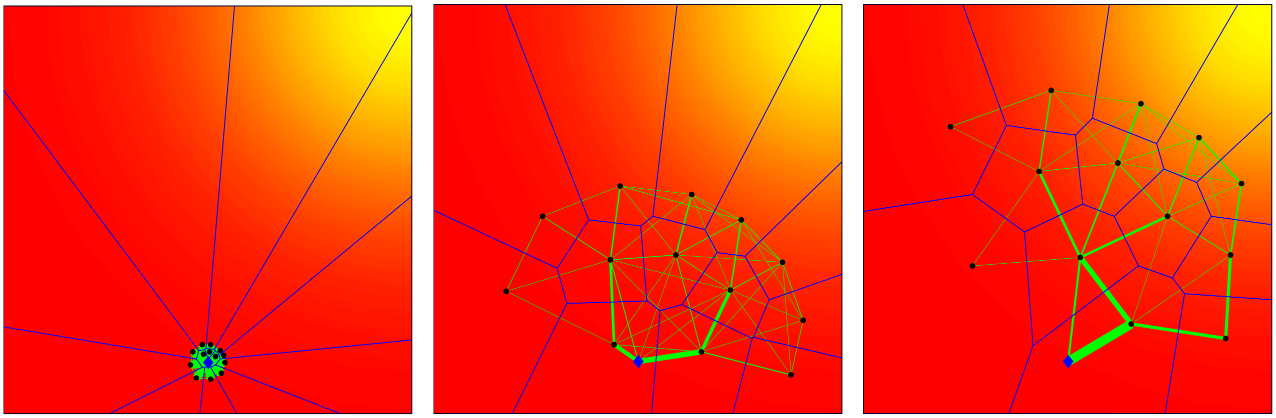
$$\begin{aligned} &\tilde{\mathcal{H}}(\mathbf{x}_i, \{\mathbf{x}_j(t_k)\}_{j \neq i}) = \\ &\mathcal{H}(\mathbf{x}(t_k)) + (\nabla_{\mathbf{x}_i(t_k)} \mathcal{H}(\mathbf{x}(t_k)))^T (\mathbf{x}_i - \mathbf{x}_i(t_k)) \\ &\quad + (\mathbf{x}_i - \mathbf{x}_i(t_k))^T H^k (\mathbf{x}_i - \mathbf{x}_i(t_k)), \end{aligned} \tag{11}$$

where H^k stands for a negative definite approximation of the Hessian of $\mathcal{H}(\mathbf{x}(t_k))$, which can be obtained with known techniques, such as the BFGS method [20] that only requires the gradient of $\mathcal{H}$. Also, we have introduced a trust-region constraint, for some $\sigma > 0$, that defines a region where the aforementioned concave and linear model are adequate approximations of the coverage objective and the communication constraint, respectively.

Rewriting (1) in discrete time, we obtain the controller for the i-th robot as

$$\mathbf{u}_i(t) = \frac{\mathbf{x}_i(t_{k+1}) - \mathbf{x}_i(t_k)}{\Delta t}, \; \forall t \in (t_k, t_{k+1}). \tag{12}$$

Remark 4.1 (Gradients): Observe that the gradients $\nabla_{\mathbf{x}_i(t_k)} \mathcal{H}$ and $\nabla_{\mathbf{x}_i(t_k)} r_i$ are required for the linearization of $\mathcal{H}$ and c_i, respectively. According to Theorem 2.2 in [2], we have



(a) Time $k = 0$

(b) Time $k = 700$

(c) Time $k = 2145$

Fig. 1. Area coverage optimization for a network consisting of $N = 13$ robots (black dots) and $K = 1$ AP (blue rhombus). Figs. 1(a) through 1(c) show the evolution of the system at different time instants. Green lines represent the communication links among the nodes. Their thickness corresponds to the rates $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$, so that thicker lines capture higher rates. A presence of a source in the upper right corner is captured by a higher density depicted in yellow.

that $\nabla_{\mathbf{x}_i(t_k)}\mathcal{H}(\mathbf{x}) = 2\int_{\mathcal{V}_i}(\mathbf{q} - \mathbf{x}_i(t_k))e^{-\|\mathbf{q}-\mathbf{x}_i(t_k)\|^2}\phi(\mathbf{q})\mathrm{d}\mathbf{q}$, where the distributed evaluation of the i-th Voronoi cell can be achieved via the algorithm presented in [1]. As for the gradient $\nabla_{\mathbf{x}_i(t_k)}r_i(\mathbf{x})$, it can be computed according to Proposition 3.1 in [14].

Remark 4.2 (Hessian): Computation of the Hessian in (11) requires only local information as the term $\mathcal{H}(\mathbf{x}(t_k))$ does not include the variable $\mathbf{x}_i(t_{k+1})$ and, therefore, it does not affect the optimization in (10).

V. Numerical Simulations

In this section we provide a simulation study of a coverage task involving a mobile robot network consisting of $N = 13$ robots and $K = 1$ AP. The area of interest is a square with side equal to 2 units of length and the density function ϕ is assumed to be a Gaussian centered at the top right corner of the square. The channel reliability is modeled by the following function

$$R(\mathbf{x}_i, \mathbf{x}_j) = \begin{cases} 1 & \text{if } \|\mathbf{x}_{ij}\| < l \\ \sum_{p=0}^{3} a_p \|\mathbf{x}_{ij}\|^p & \text{if } l < \|\mathbf{x}_{ij}\| \le u \\ 0 & \text{if } \|\mathbf{x}_{ij}\| > u \end{cases},$$

where $\|\mathbf{x}_{ij}\| = \|\mathbf{x}_i - \mathbf{x}_j\|$ and the constants a_p, $p = 0, \ldots, 3$ are chosen so that $R(\mathbf{x}_i, \mathbf{x}_j)$ is a differentiable function [12].

Fig. 1 depicts the network at different instances of its evolution along with the quality of the communication links. In this simulation study, the limits l, u are selected to be equal to 0.3 and 0.65 units, respectively. As the diameter of the region of interest is approximately 4 times the value of u, multi-hop communication is necessary in order to cover the whole area as shown in Fig. 1. In Fig. 2, the quantity $r_{out} - r_{in}$ is plotted with respect to time showing that the robots are able to maintain integrity of the communication network, as defined by equation (5).

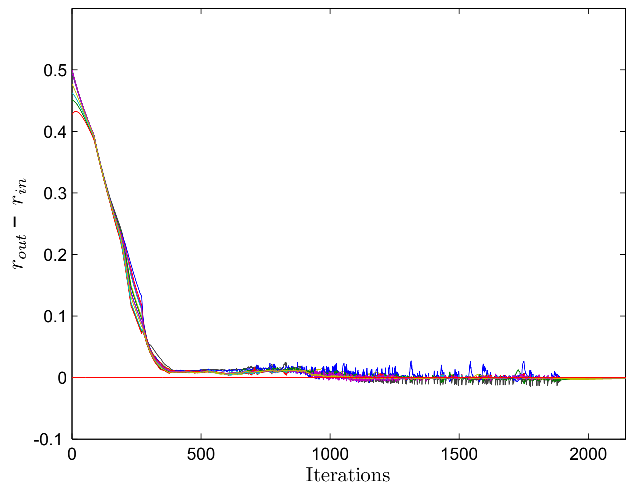


Fig. 2. Graphical depiction of the difference $r_{out} - r_{in}$ for all robots of the network.

VI. Conclusions

In this paper, we presented a distributed control scheme for maximizing the area coverage by a mobile sensor network and at the same time ensuring that packets of information are reliably relayed to a set of APs. The information generated by the sensors depended on both their sensing capabilities and the frequency at which events occur in their vicinity. Then this information was routed to the APs through a multi-hop network whose communication links modeled channel reliabilities. A hybrid scheme was proposed that decouples the optimization of the coverage objective from the control of the communication variables. Particularly, the update of the communication variables was performed periodically in the dual domain and was followed by robot mobility due to a distributed sequential concave program designed to optimize the coverage objective. Simulation studies verified the efficacy of the proposed method.

References

[1] J. Cortés, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004.

[2] J. Cortés, S. Martinez, and F. Bullo, "Spatially-distributed coverage optimization and control with limited-range interactions," *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 11, no. 4, pp. 691–719, 2005.

[3] A. Gusrialdi, T. Hatanaka, and M. Fujita, "Coverage control for mobile networks with limited-range anisotropic sensors," in *47th IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, pp. 4263–4268.

[4] B. Hexsel, N. Chakraborty, and K. Sycara, "Coverage control for mobile anisotropic sensor networks," in *IEEE International Conference on Robotics and Automation*, Shanghai, May 2011, pp. 2878–2885.

[5] L. Pimenta, V. Kumar, R. C. Mesquita, and G. Pereira, "Sensing and coverage for a network of heterogeneous robots," in *47th IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, pp. 3947–3952.

[6] C. Caicedo-Nuez and M. Zefran, "A coverage algorithm for a class of non-convex regions," in *47th IEEE Conference on Decision and Control*, Cancun, Mexico, December 2008, pp. 4244–4249.

[7] Y. Kantaros, M. Thanou, and A. Tzes, "Visibility-oriented coverage control of mobile robotic networks on non-convex regions," in *IEEE International Conference on Robotics and Automation*, Hong Kong, China, June 2014, pp. 1126–1131.

[8] M. M. Zavlanos and G. J. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Transactions on Robotics,*, vol. 23, no. 4, pp. 812–816, 2007.

[9] M. Zavlanos and G. Pappas, "Distributed connectivity control of mobile networks," *IEEE Transactions on Robotics*, vol. 24, no. 6, pp. 1416–1428, 2008.

[10] G. Notarstefano, K. Savla, F. Bullo, and A. Jadbabaie, "Maintaining limited-range connectivity among second-order agents," in *IEEE American Control Conference*, Minneapolis, MN,USA, June 2006, pp. 6–pp.

[11] M. Zavlanos, M. Egerstedt, and G. Pappas, "Graph theoretic connectivity control of mobile robot networks," *Proc. of the IEEE*, vol. 99, no. 9, pp. 1525–1540, 2011.

[12] M. M. Zavlanos, A. Ribeiro, and G. J. Pappas, "Network integrity in mobile robotic networks," *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 3–18, 2013.

[13] ——, "Mobility & routing control in networks of robots," in *49th IEEE Conference on Decision and Control*, Atlanta, GA, USA, December 2010, pp. 7545–7550.

[14] Y. Kantaros and M. M. Zavlanos, "Communication-aware coverage control for robotic sensor networks," in *IEEE 53rd Conference on Decision and Control*, Los Angeles, CA, December 2014.

[15] W. Jiang and M. Zefran, "Coverage control with information aggregation," in 52^nd *IEEE Conference on Decision and Control*, Firenze, Italy, December 2013, pp. 5421–5426.

[16] W. Li and C. G. Cassandras, "Distributed cooperative coverage control of sensor networks," in *44th IEEE Conference on Decision and Control and European Control Conference*, Seville, Spain, December 2005, pp. 2542–2547.

[17] Y. Stergiopoulos, Y. Kantaros, and A. Tzes, "Distributed Control of Mobile Sensor Networks under RF Connectivity Constraints," *International Journal of Distributed Sensor Networks*, vol. 2012, 2012.

[18] F. Aurenhammer and R. Klein, *Handbook of Computational Geometry*. Elsevier Publishing House, 1999, ch. 5: *Voronoi Diagrams*, pp. 201–290.

[19] F. Bullo, J. Cortés, and S. Martinez, *Distributed Control of Robotic Networks*. Princeton University Press, 2009.

[20] J. Nocedal and S. J. Wright, *Numerical Optimization 2nd*. Springer, 2006.