# Communication-Aware Coverage Control for Robotic Sensor Networks

Yiannis Kantaros and Michael M. Zavlanos

*Abstract*— **The purpose of this paper is to propose a control scheme to maximize area coverage and at the same time ensure reliable communication in networks of mobile robot sensors. The information that is generated at the sensors depends on the sensing capabilities of the sensors as well as on the frequency at which events occur in their vicinity, captured by appropriate probability density functions. This information is then routed to a fixed set of access points via a multi-hop network whose links model the probability that information packets are correctly decoded at their intended destinations. The proposed hybrid controller simultaneously optimizes coverage and routing of information by decoupling control in the continuous and discrete domains. The robots' motion is performed in continuous time, along the negative gradient of a cost function that combines the coverage objective and a barrier potential used to ensure satisfaction of desired communication rates. On the other hand, the communication variables are updated periodically, in discrete time, by the solution of an optimization problem, and constitute the switching signal in the continuous motion control. Simulation studies are conducted verifying the efficiency of the proposed algorithm.**

## I. Introduction

The area coverage problem is related to the development of a control plan that allows a group of mobile agents equipped with sensing and communication capabilities to spatially configure themselves in a way that maximizes the cumulative probability that events are detected in an area of interest. While the area coverage problem has recently received a lot of attention, ensuring that the collected rates of information can be efficiently relayed to a desired set of access points for subsequent processing is, to the best of our knowledge, still an open problem. In this paper, we provide a solution to this problem of joint coverage and communication control.

The literature related to coverage problems is quite extensive. In [1], the authors propose a distributed controller based on Lloyd's algorithm for sensing a convex area. In this work, it is assumed that the sensing performance degrades as the distance from the sensor increases. The case where the robots are equipped with range-limited sensors is discussed in [2]. Distributed controllers for coverage optimization have been proposed in [3] that minimize the energy needed for sensing and data processing. Coverage optimization for anisotropic sensors, whose performance depends on both the distance from the sensor and its orientation, is studied in [4], [5], while [6]–[9] discuss coverage of non-convex areas.

The area coverage problems discussed above typically ignore the requirement that the information collected by the robot sensors needs to be routed to a desired set of destinations. Introducing this capability in the system gives a new twist to the problem on the interface with communication control and networking. Most of the existing approaches to communication control of mobile robot networks employ proximity graphs to model information exchange between robots and, therefore, consider the problem of preserving graph connectivity. Such approaches involve, for example, maximization of the algebraic connectivity of the graph [10], [11], potential fields that model loss of connectivity as an obstacle in the free space [12], and distributed hybrid approaches that decompose control of the discrete graph from continuous motion of the robots [13]. Distributed algorithms for graph connectivity maintenance have also been implemented in [14], [15]. A comprehensive survey of this literature can be found in [16].

A more realistic communication model between mobile robots, compared to the above graph-theoretic approaches, is presented in [17], [18] that takes into account the routing of packets as well as desired bounds on the transmitted rates. In this model, edges in the communication graph are associated with the probability that packets delivered through the corresponding links are correctly decoded by their intended receivers. This formulation gives rise to optimization problems to determine the desired rates and routes. Related methods for the control of wireless robot networks are proposed in [19] and [20], where the wireless channels are modeled using path loss, shadowing, and multi-path fading, or evaluated using on-line techniques, respectively.

In this paper, we consider the coverage problem with the additional requirement that the information collected by the mobile robot sensors can be efficiently routed to a set of desired access points. Each agent is responsible for sensing a region and transmitting sensor measurement information to a fixed infrastructure of base stations. The rate of transmitted information depends on the quality of sensing as a function of the sensing range as well as on the on the probability that events occur in the vicinity of the sensors, captured by an appropriate probability density function over the area of interest. The underlying communication between robots follows the model proposed in [17], [18]. Our approach to joint coverage and communication control is based on decomposing the problem in the continuous and discrete domains, so that optimal coverage is achieved via continuous gradient descent on area-based coverage potential functions that respect communication constraints, while optimal communication is updated periodically via the solution of appropriate network optimization problems. In the resulting hybrid system, the communication variables constitute the switching

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signal in the continuous motion controllers. We validate our approach through non-trivial computer simulations.

A related problem that considers the minimization of the aggregate information delivered directly, in one hop, from the robots to a sink node is addressed in [21]. Multi-hop communication in the context of coverage is considered in [22] and [23]. Specifically, in [22] the objective is to minimize the energy consumption in the network, so paths are sought that ensure this minimum energy objective. In [23] a joint coverage and graph connectivity framework is developed for robots that have limited, proximity-based communication ranges. These approaches differ from the one proposed here in that we consider more realistic models of wireless communication that involve routing of information over a network of varying link reliabilities, and we also ensure desired information rates that depend on the frequency with which events occur in the sensors' vicinity.

The rest of this paper is organized as follows. Section II presents the coverage problem in the presence of communication constraints. The proposed control scheme is presented in section III while its efficiency is examined in section IV through a simulation study. Conclusive remarks are provided in the last section.

## II. Problem Formulation

Assume a team of $N$ mobile robots responsible for the sensing coverage of a convex and compact area $\mathcal{A} \subset \mathbb{R}^2$ and for the transmission of packets of information to a fixed set of $K$ access points (APs). The positions of all nodes are stacked in the vector $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_i^T, \ldots, \mathbf{x}_{N+K}^T]^T$ where $i \in \{1, ..., N\}$ for the robots and $i \in \{N+1, ..., N+K\}$ for the APs. The motion of the robots is assumed to be governed by the following first-order differential equation:

$$\dot{\mathbf{x}}_i = \mathbf{u}_i, \quad i = 1, \ldots, N, \tag{1}$$

where $\mathbf{u}_i \in \mathbb{R}^2$ stands for the control input associated with the $i$-th robot.

To achieve area coverage, each robot is equipped with an isotropic sensor whose accuracy is captured by a radially non-decreasing function that is minimum at the sensor location. In this context, a smaller value of $f$ means better accuracy. In particular, we choose $f(\mathbf{x}_i, \mathbf{q}) = \|\mathbf{q} - \mathbf{x}_i\|^2$. Moreover, let $\phi(\mathbf{q}) : \mathcal{A} \to \mathbb{R}_+$ be an integrable density function representing the probability that an event takes place at the point $\mathbf{q} \in \mathcal{A}$. Then, the coverage problem can be formulated as follows:

$$\underset{\mathbf{x}}{\text{minimize}}\ \mathcal{H}(\mathbf{x}) = \int_{\mathcal{A}} \max_{i=1,\ldots,n} f(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q}. \tag{2}$$

A common geometric approach to simplify the area cost function $\mathcal{H}$ is via the tessellation of the area of interest into subregions $\mathcal{W}_i$, $i \in \{1, ..., N\}$ according to a distance metric, and the assignment of those regions to the robots for sensing purposes. Considering that $\bigcup_{i=1}^N \mathcal{W}_i = \mathcal{A}$, this approach allows us to reformulate the coverage problem (2) as:

$$\underset{\mathbf{x},\mathcal{W}}{\text{minimize}}\ \mathcal{H}(\mathbf{x}, \mathcal{W}) = \sum_{i=1}^{N} \int_{\mathcal{W}_i} f(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q}, \tag{3}$$

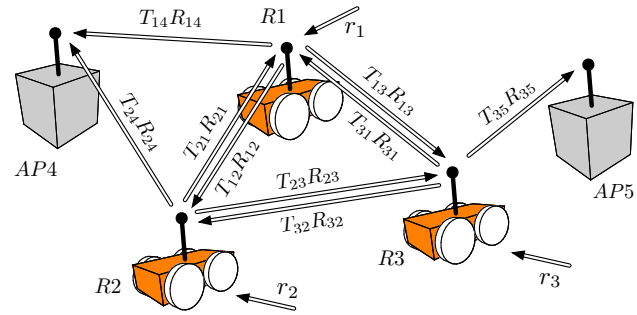



Fig. 1. Robot network consisting of two access points (APs) and three robots (Rs). Robots generate data packets at a rate $0 \leq r_i \leq 1$ and collaborate to convey this information to the APs. The position function $R(\mathbf{x}_i, \mathbf{x}_j)$ is the reliability of the channel between robots $i$ and $j$, whereas $T_{ij}$ denotes the probability that robot $i$ routes packets to robot $j$. The product $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$ is the average rate at which packets are successfully conveyed from robot $i$ to terminal $j$.

where $\mathcal{W} = \{\mathcal{W}_i\}_{i=1}^N$ denotes the collection of regions $\mathcal{W}_i$.

The problem that we address in this paper is the optimization of the objective $\mathcal{H}$ in (3), subject to communication constraints required to ensure desired information flows from the sensor robots to the access points (APs). In particular, let $R(\mathbf{x}_i, \mathbf{x}_j)$ be a link reliability metric denoting the probability that a packet transmitted by the $i$-th robot is correctly decoded by the $j$-th node. Using $R_0$ to denote the transmission rate of the terminals' radios, the effective transmission rate from $i$ to $j$ is the rate $R_0 R(\mathbf{x}_i, \mathbf{x}_j)$ at which information is successfully conveyed through this link. To simplify notation we work with normalized rates by making $R_0 = 1$. This means that rates are measured as (dimensionless) fractions of the transmission rate $R_0$. For simplicity, we also assume all robots use the same transmission rate $R_0$.

Moreover, we denote by $r_i \in [0, 1]$ the normalized average rate (information units per unit of time) at which the $i$-th robot generates information. Recalling that $R_0$ is the transmission rate of the terminals' radios, the effective rate at which information is generated at terminal $i$ is:

$$r_i(\mathbf{x}_i, \mathcal{W}_i) = R_0 \int_{\mathcal{W}_i} m(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q}, \tag{4}$$

where $m \in [0, 1]$ is any non-increasing function of the distance $\|\mathbf{q} - \mathbf{x}_i\|$ that models the probability that an event that occurs at a distance $\|\mathbf{q} - \mathbf{x}_i\|$ from the sensor is correctly captured by this sensor.[1] If a constant function $m$ is selected then the sensor quality is the same in the whole region $\mathcal{W}_i$ and $r_i$ captures the the cumulative probability that an event takes place in $\mathcal{W}_i$. Otherwise, $r_i$ takes into account the degradation of the sensing performance over the $\phi$-weighted area $\mathcal{W}_i$. A possible choice for the function $m$ is $m(\mathbf{x}_i, \mathbf{q}) = e^{-\|\mathbf{q} - \mathbf{x}_i\|^2}$.

Packets generated at the terminal $i$ are transmitted to terminal $j$ according to routing probability $T_{ij}$ representing

[1] Note that the functions $f$ and $m$ in (2) and (4), respectively, have essentially the same role. In fact, one could replace the function $f$ in (2) with $m$ and transform the minimization problem into a maximization. We choose to keep $f$ in the definition of (2) as this is a standard formulation in the coverage literature [24].

the probability that the $i$-th robot selects robot $j$ as a destination for its transmitted packets. Upon generation or arrival from another robot, packets are assumed to be stored in a queue at each robot and they leave this queue provided they are transmitted and correctly decoded by any other node $j$. Thus, the normalized rate at which packets leave the queue at the $i$-th node and are conveyed to the $j$-th node is $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$, since the transmission and the decoding process are two independent events. Packets can be conveyed by the $i$-th robot to the APs either directly if the probability $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$ for $j \in \{N+1, \ldots, N+K\}$ is reasonably large or through a multi-hop communication path; see Figure 1. Then, the average rate at which packets leave the $i$-th queue is:

$$r_i^{out} = \sum_{j=1}^{N+K} T_{ij}R(\mathbf{x}_i, \mathbf{x}_j). \tag{5}$$

Similarly, the average rate at which packets arrive at the $i$-th queue is:

$$r_i^{in} = r_i(\mathbf{x}_i, \mathcal{W}_i) + \sum_{j=1}^{N} T_{ji}R(\mathbf{x}_j, \mathbf{x}_i). \tag{6}$$

Note that the APs can only receive information which explains the upper limits in the sums of equations (5) and (6). A necessary condition to ensure that the queue at node $i$ empties infinitely often with probability one is that $r_i^{in} \leq r_i^{out}$. Therefore, packets are almost surely eventually delivered to the APs as long as

$$r_i(\mathbf{x}_i, \mathcal{W}_i) \leq \sum_{j=1}^{N+K} T_{ij}R(\mathbf{x}_i, \mathbf{x}_j) - \sum_{j=1}^{N} T_{ji}R(\mathbf{x}_j, \mathbf{x}_i) \tag{7}$$

for all $i \in \{1, \ldots, N\}$. Incorporating the set of constraints (7) as well as the probability constraint $\sum_{j=1}^{N+K} T_{ij} \leq 1$ in the optimization problem (3) we obtain the constrained coverage problem:

$$\begin{aligned} \underset{\mathbf{x},\mathbf{T},\mathcal{W}}{\text{minimize}} \quad & \mathcal{H}(\mathbf{x}, \mathcal{W}) = \sum_{i=1}^{N} \int_{\mathcal{W}_i} f(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q} \\ \text{subject to} \quad & r_i(\mathbf{x}_i, \mathcal{W}_i) \leq \sum_{j=1}^{N+K} T_{ij}R(\mathbf{x}_i,\mathbf{x}_j) - \sum_{j=1}^{N} T_{ji}R(\mathbf{x}_j,\mathbf{x}_i), \\ & \sum_{j=1}^{N+K} T_{ij} \leq 1, \quad 0 \leq T_{ij} \leq 1 \end{aligned} \tag{8}$$

where $\mathbf{T} \in \mathbb{R}^{N(N+K)}$ is the stack vector of all routing probabilities $T_{ij}$, and the constraints in (8) hold for all robots $i \in \{1, \ldots, N\}$.

Note that (8) is an optimization problem with respect to the robot positions $\mathbf{x}_i$, the routing probabilities $T_{ij}$, and the partition of the area in regions $\mathcal{W}_i$. In the absence of the constraints, it is a well known result that the objective function $\mathcal{H}$ in (8) is minimized if the partition $\mathcal{W}_i$ is chosen to be the Voronoi partition of the space [24], defined as:

*Definition 2.1 ( [25]): Voronoi diagrams* generated by a set of points located at $\{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$ is the set $\mathcal{V} = \{\mathcal{V}_1, ..., \mathcal{V}_N\}$ where $\mathcal{V}_i$ is called the *Voronoi cell* of node $i$ and contains the points that are closer to it than to any other node:

$$\mathcal{V}_i = \{\mathbf{q} \in \mathcal{A} \mid \|\mathbf{q} - \mathbf{x}_i\| \leq \|\mathbf{q} - \mathbf{x}_j\|, \ \forall j \neq i\}.$$

On the other hand, in the presence of the constraints, the Voronoi regions are not necessarily feasible for (8) and, therefore, the feasible optimal partition for the constrained problem (8) is in general different from the Voronoi partition. However, if we are able to ensure feasibility of the Voronoi partition, then this partition will be optimal for (8). In our problem, this is possible by appropriately selecting the routing probabilities $T_{ij}$. In fact, we replace the partition $\mathcal{W}$ in (8) by the Voronoi partition $\mathcal{V}$, and then solve for routes $T_{ij}$ that satisfy the constraints. This gives rise to the following problem:

$$\begin{aligned} \underset{\mathbf{x},\mathbf{T}}{\text{minimize}} \quad & \mathcal{H}(\mathbf{x}, \mathcal{V}) = \sum_{i=1}^{N} \int_{\mathcal{V}_i} f(\mathbf{x}_i, \mathbf{q})\phi(\mathbf{q})\mathrm{d}\mathbf{q} \\ \text{subject to} \quad & r_i(\mathbf{x}_i, \mathcal{V}_i) \leq \sum_{j=1}^{N+K} T_{ij}R(\mathbf{x}_i,\mathbf{x}_j) - \sum_{j=1}^{N} T_{ji}R(\mathbf{x}_j,\mathbf{x}_i), \\ & \sum_{j=1}^{N+K} T_{ij} \leq 1, \quad 0 \leq T_{ij} \leq 1 \end{aligned} \tag{9}$$

where again the constraints in (9) hold for all robots $i \in \{1, \ldots, N\}$. Assume now that the network is initially deployed so that the constraints (7) are satisfied. Then, in this paper we seek a solution to the following problem:

*Problem 1:* Determine robot positions $\mathbf{x}_i$ and routes $\{T_{ij}\}_{j=1}^{N+K}$ such that coverage is optimized and reliable communication with the APs is guaranteed, as per the solution of problem (9).

## III. Coverage and Routing Control

In this section, a hybrid control scheme is proposed for solving the non-linear optimization problem (9). More specifically, for fixed values of the routing probabilities $T_{ij}$, we design continuous motion controllers $\mathbf{u}_i$ for the robots to optimize the coverage objective in (3), while maintaining feasibility of the routing constraints (7). As the robots move, the generated rates of information $r_i$ and the channel reliabilities $R_{ij}$ change, so we periodically update the routing probabilities to account for these changes. The new routes $T_{ij}$ are found by solving the feasibility problem (7) for the current values of $r_i$ and $R_{ij}$. The routing variables $T_{ij}$ constitute the switching signal in the continuous motion controllers (1).

In particular, let $\{t_k\}_{k=0}^{\infty}$ denote a sequence of time instances at which the routing probabilities $T_{ij}$ are updated and define the switching signal affecting the controller of robot $i$ by

$$\sigma_i(t_k) = \{T_{ij}(t_k)\}_{j=1}^{N+K} \cup \{T_{ji}(t_k)\}_{j=1}^{N}. \tag{10}$$

**Algorithm 1** Simultaneous coverage and routing control

**Require:** Initial position $\mathbf{x}_i(t_0)$ for all robots $i$;

Compute the routing probabilities $T_{ij}(t_0)$ by solving the feasibility problem of constraints (7) for all $i$;
**for** $k = 0$ to $\infty$ **do**
  Continuously move every robot $i$ according to the closed loop system (1)–(12) until time $t_{k+1}$;
  When $t = t_{k+1}$, compute the new routing probabilities $T_{ij}(t_{k+1})$ by solving the feasibility problem of constraints (7) for all $i$;
  Set $k := k + 1$;
**end for**

Then, according to the proposed scheme the motion of the robot $i$ is governed by:

$$\dot{\mathbf{x}}_i = \mathbf{u}_i(\mathbf{x}(t), \sigma_i(t_k)), \quad \forall t \in [t_k, t_{k+1}). \tag{11}$$

To optimize coverage while ensuring feasibility of the constraints (7) as the robots move, we choose the controller $\mathbf{u}_i(\mathbf{x}(t), \sigma_i(t_k))$ to be the negative gradient:

$$\mathbf{u}_i(\mathbf{x}(t), \sigma_i(t_k)) = -\nabla_{\mathbf{x}_i}\mathcal{H}(\mathbf{x}(t)) - \epsilon\nabla_{\mathbf{x}_i}\mathcal{C}_i(\mathbf{x}(t), \sigma_i(t_k)) \tag{12}$$

for $\epsilon > 0$ a gain, where

$$\mathcal{C}_i(\mathbf{x}(t), \sigma_i(t_k)) = \Bigg[\Bigg(\sum_{j=1}^{N+K} T_{ij}(t_k)R(\mathbf{x}_i(t), \mathbf{x}_j(t))\Bigg)^2 - \Bigg(\sum_{j=1}^{N} T_{ji}(t_k)R(\mathbf{x}_j(t), \mathbf{x}_i(t)) + r_i(\mathbf{x}_i(t), \mathcal{V}_i(t))\Bigg)^2\Bigg]^{-1} \tag{13}$$

is a *barrier potential function* that grows unbounded when the constraint (7) for robot $i$ tends to become violated. This construction ensures satisfaction of (7) for all time $t \in [t_k, t_{k+1})$ when the controller (12) is active. The integrated hybrid system for joint coverage and routing control is described in the Algorithm 1.

Observe that the computation of the controller (12) requires the gradients $\nabla_{\mathbf{x}_i}\mathcal{H}$ and $\nabla_{\mathbf{x}_i}r_i$. As shown in [1], the gradient of the function $\mathcal{H}$ is given by

$$\nabla_{\mathbf{x}_i}\mathcal{H} = 2M_{\mathcal{V}_i}\left(\mathbf{x}_i - \mathbf{C}_{\mathcal{V}_i}\right), \tag{14}$$

where $M_{\mathcal{V}_i} = \int_{\mathcal{V}_i}\phi(\mathbf{q})\mathrm{d}\mathbf{q}$ and $\mathbf{C}_{\mathcal{V}_i} = \frac{1}{M_{\mathcal{V}_i}}\int_{\mathcal{V}_i}\mathbf{q}\phi(\mathbf{q})\mathrm{d}\mathbf{q}$ denote the mass and the centroid of the Voronoi cell $\mathcal{V}_i$ with density function $\phi$, respectively. To obtain the gradient of $r_i$ we show the following result:

*Proposition 3.1:* Given a differentiable function $m$ and a density function $\phi$, the gradient of the function $r_i$ defined in (4) can be expressed as follows:

$$\nabla_{\mathbf{x}_i}r_i(\mathbf{x}_i, \mathcal{V}_i) = \int_{\mathcal{V}_i}\frac{\partial m(\mathbf{q}, \mathbf{x}_i)}{\partial \mathbf{x}_i}\phi(\mathbf{q})\mathrm{d}\mathbf{q} + \sum_{e=1}^{E_i}\int_{(\mathbf{q}_{i,1}^e, \mathbf{q}_{i,2}^e)}\frac{\mathbf{q} - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|}m(\mathbf{q}, \mathbf{x}_i)\phi(\mathbf{q})\mathrm{d}\mathbf{q} \tag{15}$$

where $E_i$ is the number of edges that constitute the polygonal boundary $\partial\mathcal{V}_i$ of the Voronoi cell $\mathcal{V}_i$ and $(\mathbf{q}_{i,1}^e, \mathbf{q}_{i,2}^e)$ denotes the $e$-th edge in this boundary with end points $\mathbf{q}_{i,1}^e$ and $\mathbf{q}_{i,2}^e$, i.e., $\partial\mathcal{V}_i = \{(\mathbf{q}_{i,1}^1, \mathbf{q}_{i,2}^1), \ldots, (\mathbf{q}_{i,1}^{E_i}, \mathbf{q}_{i,2}^{E_i})\}$.

*Proof:* Applying the the Leibniz integral rule [26] to the expression for $r_i$ in (4), we have:

$$\begin{aligned}\nabla_{\mathbf{x}_i}r_i &= \frac{\partial}{\partial \mathbf{x}_i}\int_{\mathcal{V}_i} m(\mathbf{q}, \mathbf{x}_i)\phi(\mathbf{q})\mathrm{d}\mathbf{q} \\ &= \int_{\mathcal{V}_i}\frac{\partial m(\mathbf{q}, \mathbf{x}_i)}{\partial \mathbf{x}_i}\phi(\mathbf{q})\mathrm{d}\mathbf{q} + \int_{\partial\mathcal{V}_i}\mathbf{n}_i^T\frac{\partial \mathbf{q}}{\partial \mathbf{x}_i}m(\mathbf{q}, \mathbf{x}_i)\phi(\mathbf{q})\mathrm{d}\mathbf{q},\end{aligned} \tag{16}$$

where $\mathbf{n}_i = \frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|}$ is the normal vector pointing outwards of the edge of Voronoi cell $\mathcal{V}_i$ associated with the Delaunay neighbors $i$ and $j$.[2] Let $(\mathbf{q}_{i,1}^e, \mathbf{q}_{i,2}^e)$ denote the $e$-th edge in the boundary $\partial\mathcal{V}_i$. Then, the points $\mathbf{q}$ that belong to the $e$-th edge lie on the line described by the equation:

$$\mathbf{q} = \frac{\mathbf{x}_i + \mathbf{x}_j}{2} + a_{ij}\mathbf{C}_{ij}(\mathbf{x}_j - \mathbf{x}_i), \tag{17}$$

where $\mathbf{C}_{ij}$ is the skew symmetric rotation matrix

$$\mathbf{C}_{ij} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

so that $\mathbf{C}_{ij}(\mathbf{x}_j - \mathbf{x}_i)$ is perpendicular to $(\mathbf{x}_j - \mathbf{x}_i)$, and $a_{ij} \in \mathbb{R}$ is a scalar.

Taking the partial derivative of (17) with respect to $\mathbf{x}_i$ yields:

$$\frac{\partial \mathbf{q}}{\partial \mathbf{x}_i} = \frac{1}{2}\mathbf{I} - a_{ij}\mathbf{C}_{ij}, \tag{18}$$

where $\mathbf{I}$ stands for the identity matrix. Multiplying (18) from the left by $\mathbf{n}_i^T$, we have:

$$\begin{aligned}\mathbf{n}_i^T\frac{\partial \mathbf{q}}{\partial \mathbf{x}_i} &= \frac{1}{2}\mathbf{n}_i^T - a_{ij}\mathbf{n}_i^T\mathbf{C}_{ij} \\ &= \frac{1}{2}\frac{(\mathbf{x}_j - \mathbf{x}_i)^T}{\|\mathbf{x}_j - \mathbf{x}_i\|} + a_{ij}\frac{(\mathbf{x}_j - \mathbf{x}_i)^T}{\|\mathbf{x}_j - \mathbf{x}_i\|}\mathbf{C}_{ij}^T \\ &= \frac{1}{2}\frac{(\mathbf{x}_j - \mathbf{x}_i)^T}{\|\mathbf{x}_j - \mathbf{x}_i\|} + \frac{1}{\|\mathbf{x}_j - \mathbf{x}_i\|}\left(\mathbf{q}^T - \frac{(\mathbf{x}_i + \mathbf{x}_j)^T}{2}\right) \\ &= \frac{\mathbf{q}^T - \mathbf{x}_i^T}{\|\mathbf{x}_j - \mathbf{x}_i\|},\end{aligned} \tag{19}$$

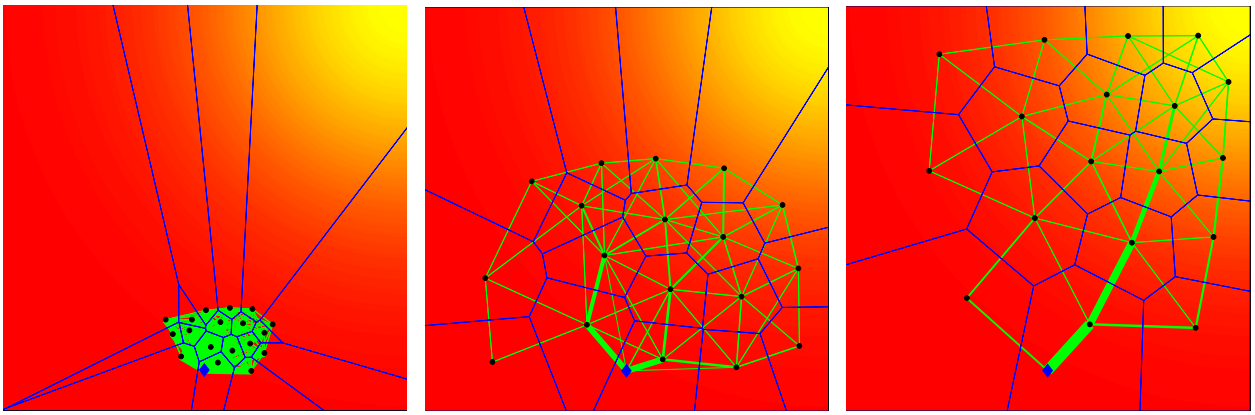
where in the third equality of (19), we have substituted the term $a_{ij}(\mathbf{x}_j - \mathbf{x}_i)^T\mathbf{C}_{ij}^T$ from (17).

Substituting equation (19) into (16) yields:

$$\nabla_{\mathbf{x}_i}r_i = \int_{\mathcal{V}_i}\frac{\partial m(\mathbf{q}, \mathbf{x}_i)}{\partial \mathbf{x}_i}\phi(\mathbf{q})\mathrm{d}\mathbf{q} + \int_{\partial\mathcal{V}_i}\frac{\mathbf{q} - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|}m(\mathbf{q}, \mathbf{x}_i)\phi(\mathbf{q})\mathrm{d}\mathbf{q}. \tag{20}$$

Finally, taking into account the decomposition of the boundary of the Voronoi cell into edges $\partial\mathcal{V}_i = \{(\mathbf{q}_{i,1}^1, \mathbf{q}_{i,2}^1), \ldots, (\mathbf{q}_{i,1}^{E_i}, \mathbf{q}_{i,2}^{E_i})\}$, equation (20) gives (15), which completes the proof of the proposition. ■

[2]Two nodes are called Delaunay neighbors if they share an edge of their corresponding Voronoi cells.



(a) Time $t = 0$ (b) Time $t = 1.5$ (c) Time $t = 3$

Fig. 2. Evolution of a communication network consisting of $N = 18$ robots (black dots) and $K = 1$ AP (blue rhombus) during an area coverage task. Figures 2(a) through 2(c) show the evolution of the system at different time instants. Green lines represent the communication links among the nodes. Their thickness depends on the value of $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$, i.e., thicker lines capture higher values. A presence of a source in the upper right corner of the area is captured by a higher density depicted in yellow.

## IV. Simulation Studies

In this section we provide a simulation study of a mobile robot network consisting of $N = 18$ robots and $K = 1$ AP. The area of interest is defined as a $2 \times 2$ square and the density function $\phi$ is assumed to be a Gaussian centered at $(2, 2)$. The function $m$ in (4) is selected to be equal to $e^{-\|\mathbf{x}_i - \mathbf{q}\|^2}$ and the channel reliability is modeled by the following function:

$$R(\mathbf{x}_i, \mathbf{x}_j) = \begin{cases} 1 & \text{if } \|\mathbf{x}_{ij}\| < l \\ \sum_{p=0}^{3} a_p \|\mathbf{x}_{ij}\|^p & \text{if } l < \|\mathbf{x}_{ij}\| \leq u \\ 0 & \text{if } \|\mathbf{x}_{ij}\| > u \end{cases},$$

where $\|x_{ij}\| = \|\mathbf{x}_i - \mathbf{x}_j\|$ and the constants $a_p$, $p = 0, \ldots, 3$ are chosen so that $R(\mathbf{x}_i, \mathbf{x}_j)$ is a differentiable function [17].

Figure 2 depicts the network at different instances of its evolution along with the quality of the communication links. In this simulation study, the limits $l, u$ are selected to be equal to 0.3 and 0.6 units, respectively. As the diameter of the region of interest is approximately 4 times the value of $u$, multi-hop communication is necessary in order to cover the whole area. The rates of information transmitted over the communication links are captured by the quantities $T_{ij}R(\mathbf{x}_i, \mathbf{x}_j)$ as described in Section II. In Figure 2, we see that coverage is achieved, while multi-hop communication paths are established with the AP.

In Figure 3, the quantity $r_{out} - r_{in}$ is plotted with respect to time showing that the routing constraints (7) are satisfied through the network evolution and in Fig 4 the monotonic evolution of the coverage objective is illustrated. Moreover, in Figure 5, we show the average rate at which every robot generates information, where the higher rates correspond to robots in the upper corner in Figure 2(c), since they are close to the source.

## V. Conclusions

In this paper, a first centralized approach to the simultaneous coverage and communication control problem was

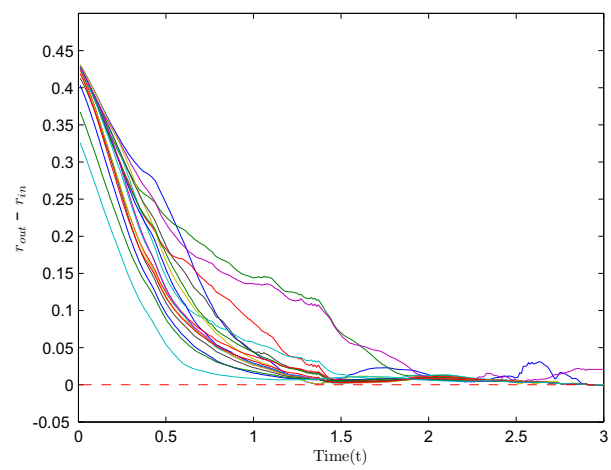



Fig. 3. Graphical depiction of the difference $r_{out} - r_{in}$ for all robots of the network.

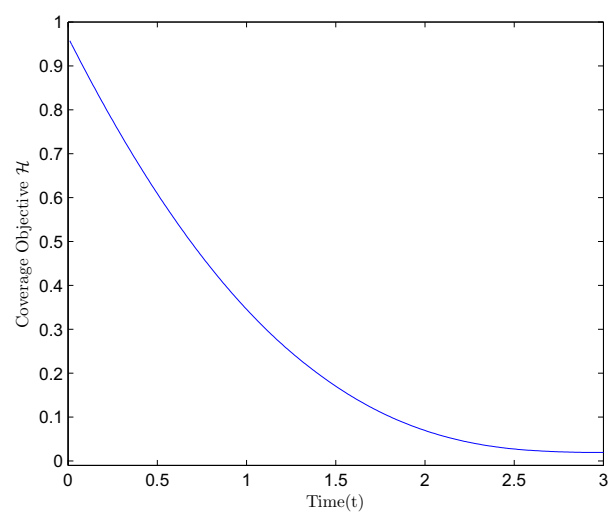



Fig. 4. Evolution of the coverage objective function $\mathcal{H}$.

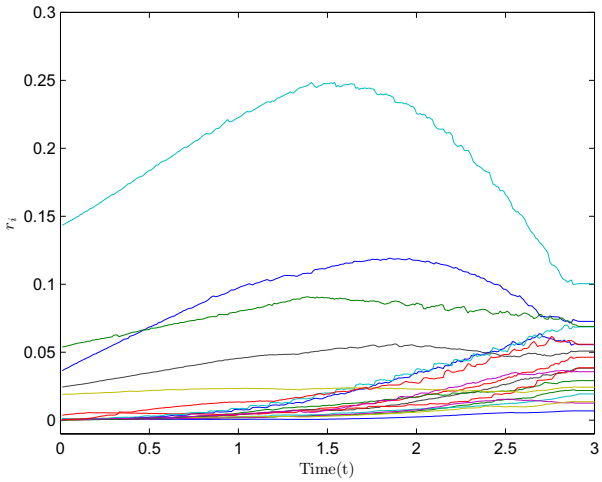



Fig. 5. Graphical representation of the average rate $r_i$ for all robots of the network.

presented. The robots were responsible for sensing a convex area and for reliably relaying packets of information that depend on their sensing capabilities and a density function defined over the area of interest, to a set of APs. The routing of packets was performed through a multi-hop network modeled so that its communication links represent channel reliabilities. In our approach, the coverage and the routing problem were addressed jointly, leading to a hybrid system. Particularly, the motion control was performed in the continuous time domain ensuring coverage optimization and satisfaction of the communication constraints, while communication control was implemented at discrete time instances through optimization of the routing variables that constituted the switching signal in the continuous robot motion. Simulation studies illustrated the efficacy of the proposed control scheme.

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