Flocking while Preserving Network Connectivity

Michael M. Zavlanos, Ali Jadbabaie and George J. Pappas

Abstract— **Coordinated motion of multiple agents raises fundamental and novel problems in control theory and robotics. In particular, in applications such as consensus seeking or flocking by a group of mobile agents, a great new challenge is the development of robust distributed motion algorithms that can always achieve the desired coordination. In this paper, we address this challenge by embedding the requirement for connectivity of the underlying communication network in the controller specifications. We employ double integrator models for the agents and design nearest neighbor control laws, based on potential fields, that serve a twofold objective. First, they contribute to velocity alignment in the system and second, they regulate switching among different network topologies so that the connectivity requirement is always met. Collision avoidance among neighboring agents is also ensured and under the assumption that the initial network is connected, the overall system is shown to asymptotically flock for all initial conditions. In particular, it is shown that flocking is achieved even in sparse communication networks where connectivity is more prone to failure. We conclude by illustrating a class of interesting problems that can be achieved while preserving connectivity.**

I. Introduction

Over the past few years, the problem of coordinated motion and cooperative control of multiple autonomous agents has received a considerable amount of attention. From ecology and evolutionary biology to social sciences, and from systems and control theory to complexity theory, statistical physics, and computer graphics, efforts have been made towards a better understanding of how a group of moving objects such as flocks of birds, schools of fish, crowds of people can perform collective tasks without centralized coordination.

In ecology and theoretical biology, such problems have been studied in the context of animal aggregation and social cohesion [1] and much research has focused in mimicking the observed social aggregation phenomena using computer simulation [2]. On the other hand, flocking and schooling behavior was recently addressed in the context of self organization of systems of self-propelled particles [3] in the fields of statistical physics and complexity theory.

In control theory and robotics, flocking and schooling behavior naturally arises in problems involving cooperative control of autonomous robots, unmanned vehicles, and multi-agent systems. A nonexhaustive list of references include [4] – [26]. A frequently used model for flocking and coordination is proposed in [3], where the agents are assumed to have a common and constant speed and updating of their headings is based on a simple averaging rule involving nearest neighbors only. The model is in discrete time and involves discontinuous switches due to changes in the neighborhood topology resulting from the agents' motion. Stability of this model was studied in [24], where it was shown that for any sequence of *jointly connected* nearest neighbor graphs the headings of all agents converge to a common value. Extensions of this model to vision-based scenarios were also studied in [25]. More recently, flocking in multi-agent systems has also been studied for dynamic point-mass models [26]. Inspired by [2] the authors in [26] propose distributed control laws that guarantee alignment, separation and cohesion of the group, capturing hence, the essence of the model proposed in [2].

To the best of our knowledge, consensus or flocking results so far, critically rely on the assumption that the underlying communication network is either connected for all time [23], [26] or is *jointly connected* over infinite sequences of bounded time intervals [24]. However, one can easily imagine scenarios where the topology of the communication graph is sparse and hence, losing even a few links can destroy connectivity. In the spirit of *robust* algorithms for multi-agent flocking, in this paper, we propose a distributed control framework that *simultaneously* addresses the desired velocity alignment as well as the connectivity requirement of the underlying network, necessary for alignment. As in [26], we employ double integrator models for the agents and design distributed control laws, based on potential fields, that achieve velocity alignment and ensure that switching occurs among connected network topologies, by maintaining existing links in the network. The network topology is described by proximity graphs, slightly modified to include a *hysteresis* in link additions. It is due to this novel idea that regulating the network topology becomes possible. Under the assumption that the initial network is connected, the overall system is shown to asymptotically flock for all initial conditions, while collision avoidance can also be guaranteed. Our approach is finally illustrated through a class of interesting problems that can be achieved while preserving connectivity.

The rest of this paper is organized as follows. In Section II we define the multi-agent flocking problem, as well as necessary tools from graph theory. In Section III we propose the distributed control laws and show that by means of regulating switching among different network topologies, they guarantee both collision avoidance and velocity alignment for all initial conditions. Our approach is finally illustrated in Section IV, where comparisons with other approaches in the literature demonstrate its robust nature.

This work is partially supported by ARO MURI SWARMS Grant W911NF-05-1-0219 and the NSF ITR Grant 0324977.

Michael M. Zavlanos, Ali Jadbabaie and George J. Pappas are with GRASP Laboratory, Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, PA 19104, USA {zavlanos,jadbabai,pappasg}@grasp.upenn.edu

II. Problem Formulation

Consider n mobile agents in $\mathbb{R}^m$ and let the dynamics of agent i be described by a double integrator,

$$\begin{aligned}\dot{x}_i(t) &= v_i(t)\\ \dot{v}_i(t) &= u_i(t)\end{aligned} \tag{1}$$

where $x_i(t), v_i(t) \in \mathbb{R}^m$ denote the position and velocity vectors of agent i at time t, respectively, and $u_i(t) \in \mathbb{R}^m$ is a control vector to be determined. The goal of this paper is to determine control inputs $u_i(t) \in \mathbb{R}^m$ for all agents i so that the group *flocks* in the following sense.

Definition 2.1 (Flocking): A group of mobile agents is said to (asymptotically) flock, when all agents attain the same velocity vector, distances between the agents are stabilized and no collisions among them occur.

Stability analysis of the group of agents relies on several results from algebraic graph theory [27]. In particular, in view of the multi-agent dynamics described in system (1), we can define a *dynamic graph* $\mathcal{G}(t)$ as follows.

Definition 2.2 (Dynamic Graphs): We call $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ a dynamic graph consisting of a set of vertices $\mathcal{V} = \{1, \ldots, n\}$ indexed by the set of agents and a time varying set of links $\mathcal{E}(t) = \{(i,j) \mid i,j \in \mathcal{V}\}$ such that, for any $0 < r < R$,

- if $0 < \|x_i(t) - x_j(t)\|_2 < r$ then, $(i,j) \in \mathcal{E}(t)$,
- if $R \leq \|x_i(t) - x_j(t)\|_2$ then, $(i,j) \notin \mathcal{E}(t)$.

Dynamic graphs $\mathcal{G}(t)$ such that $(i,j) \in \mathcal{E}(t)$ if and only if $(j,i) \in \mathcal{E}(t)$ are called *undirected* and consist the main focus of this paper. Moreover, any vertices i and j of an undirected graph $\mathcal{G}(t)$ that are joined by a link $(i,j) \in \mathcal{E}(t)$, are called adjacent or neighbors at time t and are denoted by $i \sim j$. Clearly, Definition 2.2 specifies the switching process among graphs (Figure 1), while the *hysteresis* introduced in creation of new links consists the key idea that enables our approach (see Proposition 3.1). Note also that *collision avoidance* between adjacent agents i and j in $\mathcal{G}(t)$ is implied due to the requirement that $0 < \|x_i(t) - x_j(t)\|_2$.[1]

A topological invariant of graphs that is of particular interest for the purposes of this paper is graph connectivity.

Definition 2.3 (Graph Connectivity): We say that a dynamic graph $\mathcal{G}(t)$ is connected at time t if there exists a path, i.e., a sequence of distinct vertices such that consecutive vertices are adjacent, between any two vertices in $\mathcal{G}(t)$.

Hence, the problem addressed in this paper can be formally stated as follows.

Problem 1 (Flocking): Given the set of connected graphs $\mathcal{C}_n$ on n vertices, determine distributed control laws $u_i(t)$ for all agents i so that if $\mathcal{G}(t_0) \in \mathcal{C}_n$, then $\mathcal{G}(t) \in \mathcal{C}_n$ for all time $t \geq t_0$, all agent velocities asymptotically become the same and collisions among them are always avoided.

Note that unlike previous approaches to the problem, that critically rely on the *assumption* that the network $\mathcal{G}(t)$ is connected for all time [26] or infinitely often [24], here we only require that $\mathcal{G}(t)$ is *initially* connected. Then, our approach to Problem 1 critically relies in rendering the set $\mathcal{C}_n$ an invariant of motion for system (1). We achieve this goal by choosing an equivalent formulation of the problem using the algebraic representation of the dynamic graph $\mathcal{G}(t)$. In particular, the structure of any dynamic graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ can be equivalently represented by a dynamic *Laplacian* matrix,

$$L(t) = \Delta(t) - A(t) \tag{2}$$

where $A(t) = (a_{ij}(t))$ corresponds to the *Adjacency matrix* of the graph $\mathcal{G}(t)$, which is such that $a_{ij}(t) = 1$ if $i \sim j \in \mathcal{E}(t)$ and $a_{ij}(t) = 0$ otherwise and $\Delta(t) = \text{diag}\big(\sum_{j=1}^n a_{ij}(t)\big)$ denotes the *Valency matrix*.[2] Note that for undirected graphs, the Adjacency matrix is a symmetric matrix and hence, so is the Laplacian matrix. The spectral properties of the Laplacian matrix are closely related to graph connectivity. In particular, we have the following lemma.

Lemma 2.4 ([27]): Let $\lambda_1(L(t)) \leq \lambda_2(L(t)) \leq \cdots \leq \lambda_n(L(t))$ be the ordered eigenvalues of the Laplacian matrix $L(t)$. Then, $\lambda_1(L(t)) = 0$ for all t, with corresponding eigenvector $\mathbf{1}$, i.e., the vector of all entries equal to 1. Moreover, $\lambda_2(L(t)) > 0$ if and only if $\mathcal{G}(t)$ is connected.

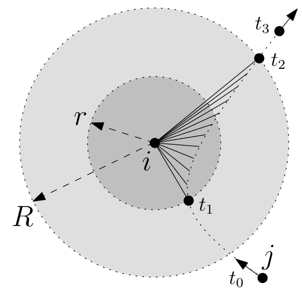


Fig. 1. Link (solid lines) dynamics according to Definition 2.1.

III. Multi-Agent Coordination

Given any dynamic graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, let $\mathcal{N}_i(t) = \{j \mid (i,j) \in \mathcal{E}(t)\}$ denote the neighbors of agent i at time t and for all i define the set of control laws,

$$u_i = -\sum_{j \in \mathcal{N}_i(t)} (v_i - v_j) - \sum_{j \in \mathcal{N}_i(t)} \nabla_{x_i} V_{ij} \tag{3}$$

where the first term in the right hand side of (3) corresponds to the desired velocity alignment, while the second term corresponds to a vector in the direction of the negated gradient of an artificial potential function,

$$V_{ij}(x_{ij}) = \frac{1}{\|x_{ij}\|_2^2} + \frac{1}{R^2 - \|x_{ij}\|_2^2}, \quad \|x_{ij}\|_2 \in (0, R)$$

with $x_{ij} = x_i - x_j$, which allows both collision avoidance and maintaining links in the network (Figure 2).[3] Note that V_{ij} grows unbounded when $\|x_{ij}\|_2 \to R^-$, hence the significance of the hysteresis in our model.

[1] Dynamic graphs $\mathcal{G}(t)$ as in Definition 2.1, are sometimes also called *proximity graphs*.

[2] Since we do not allow self-loops, we define $a_{ii}(t) = 0$ for all $i = 1, 2, \ldots, n$.

[3] For distributed coordination protocols that also allow link deletions we refer the reader to [22].

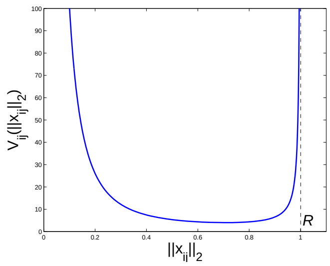


Fig. 2. Plot of the potential $V_{ij}(x_{ij})$ for $R = 1$.

As discussed in Section II, the topology of the network may change over time, hence, the system dynamics (1) under control inputs (3) result in a *switching* dynamical system. Let t_p for $p = 1, 2, \ldots$ denote the switching times when the topology of $\mathcal{G}(t)$ changes, and define a *switching signal* $\mathcal{G}(t) : [t_0, \infty) \to \mathcal{C}_n$ associated with connected graphs, according to Problem 1.[4] We, then, have the following result.

Proposition 3.1: Assume the closed loop system (1)-(3). Then, for any pair of switching times $t_p < t_q$ the switching signal $\mathcal{G}(t)$ satisfies $\mathcal{E}(t_p) \subseteq \mathcal{E}(t_q)$.

Proof: Let $t_1, t_2, \ldots$ denote the sequence of switching times and let $\mathbf{v} = [v_1^T \ \ldots \ v_n^T]^T$ and $\mathbf{u} = [u_1^T \ \ldots \ u_n^T]^T$ denote the stack vectors of all agent velocity vectors $v_i \in \mathbb{R}^m$ and control signals $u_i \in \mathbb{R}^m$, respectively, and consider the dynamical system,

$$\dot{\mathbf{x}} = B_K \mathbf{v}$$
$$\dot{\mathbf{v}} = \mathbf{u}$$

where B_K is the incidence matrix of the complete graph. Define, further, the function $V_{\mathcal{G}} : D_{\mathcal{G}} \times \mathbb{R}_+^{mn} \to \mathbb{R}_+$ such that,[5]

$$V_{\mathcal{G}} = \frac{1}{2}\Big(\|\mathbf{v}\|_2^2 + \sum_{i=1}^n V_i\Big)$$

where $V_i = \sum_{j \in \mathcal{N}_i} V_{ij}$ and $D_{\mathcal{G}} = \{\mathbf{x} \in \mathbb{R}^{mn(n-1)} | \|x_{ij}\|_2 \in (0, R) \ \forall \ (i,j) \in \mathcal{E}\}$. For any $c > 0$, let $\Omega_{\mathcal{G}} = \{(\mathbf{x}, \mathbf{v}) \in D_{\mathcal{G}} \times \mathbb{R}_+^{mn} | V_{\mathcal{G}} \leq c\}$ denote the level sets of $V_{\mathcal{G}}$ and observe that,

$$\dot{V}_{\mathcal{G}} = \frac{1}{2}\sum_{i=1}^n \dot{V}_i - \sum_{i=1}^n v_i^T \Big(\sum_{j \in \mathcal{N}_i}(v_i - v_j) + \nabla_{x_i} V_i\Big)$$

Moreover,

$$\begin{aligned}
\frac{1}{2}\sum_{i=1}^n \dot{V}_i &= \frac{1}{2}\sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \dot{x}_{ij}^T \nabla_{x_{ij}} V_{ij} \\
&= \frac{1}{2}\sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \Big(\dot{x}_i^T \nabla_{x_{ij}} V_{ij} - \dot{x}_j^T \nabla_{x_{ij}} V_{ij}\Big) \\
&= \frac{1}{2}\sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \Big(\dot{x}_i^T \nabla_{x_i} V_{ij} + \dot{x}_j^T \nabla_{x_j} V_{ij}\Big) \\
&= \sum_{i=1}^n \sum_{j \in \mathcal{N}_i} \dot{x}_i^T \nabla_{x_i} V_{ij} = \sum_{i=1}^n \dot{x}_i^T \nabla_{x_i} V_i
\end{aligned}$$

[4]Note that $\mathcal{G}(t)$ is also a map from the real time-line to the set of graphs.

[5]We denote by $\mathbb{R}_+$ the set $[0, \infty)$.

by symmetry of the functions V_{ij}. Hence,

$$\begin{aligned}
\dot{V}_{\mathcal{G}} &= \sum_{i=1}^n v_i^T \nabla_{x_i} V_i - \sum_{i=1}^n v_i^T \Big(\sum_{j \in \mathcal{N}_i}(v_i - v_j) + \nabla_{x_i} V_i\Big) \\
&= -\sum_{i=1}^n v_i^T \sum_{j \in \mathcal{N}_i}(v_i - v_j) = -\mathbf{v}^T (L_{\mathcal{G}} \otimes I_m)\mathbf{v} \leq 0
\end{aligned}$$

where $\otimes$ denotes the Kronecker product between matrices. Clearly, $\dot{V}_{\mathcal{G}}$ is always nonpositive, by positive semidefiniteness of the Laplacian matrix $L_{\mathcal{G}}$. Hence, for any signal $\mathcal{G}$, the level sets $\Omega_{\mathcal{G}}$ are positively invariant, implying that for any $(i,j) \in \mathcal{E}$, V_{ij} remains bounded. On the other hand, if for some $(i,j) \in \mathcal{E}$, $\|x_{ij}\| \to R$, then $V_{ij}(x_{ij}) \to \infty$. Thus, by continuity of $V_{\mathcal{G}}$ in $D_{\mathcal{G}}$, it follows that $\|x_{ij}\| < R$, for all $(i,j) \in \mathcal{E}$ and $t \in [t_p, t_{p+1})$. In other words, all links in $\mathcal{G}$ are maintained between switching times, which implies that $\mathcal{E}(t_p) \subseteq \mathcal{E}(t_{p+1})$. Applying recursively this argument completes the proof. A similar argument for the case where $\|x_{ij}\|_2 \to 0$ can be used to establish collision avoidance. Note finally, that the condition $0 < r < R$ in Definition 2.2 ensures that if a link $(i,j) \notin \mathcal{E}$ is added to $\mathcal{E}$, then the associated potential V_{ij} is bounded and hence, so is the new potential $V_{\mathcal{G}}$. This observation allows us to define level sets of the potentials $V_{\mathcal{G}}$. ■

Proposition 3.1 clearly implies that if $\mathcal{G}(t_0) \in \mathcal{C}_n$, then the switching signal will satisfy $\mathcal{G}(t) \in \mathcal{C}_n$ for all time $t \geq t_0$. In particular, we have the following corollary.

Corollary 3.2: Under control law (3), the total number of switching times of system (1) is finite.

Proof: Just note that, by Proposition 3.1, the size of the set of links $|\mathcal{E}(t)|$ forms an increasing sequence and,

$$\sup_{t \geq t_0}\{|\mathcal{E}(t)| - |\mathcal{E}(t_0)|\} = \frac{n(n-1)}{2} - (n-1)$$

where $n-1$ corresponds to the number of links in $\mathcal{G}(t_0)$ if it is minimally connected, i.e., if it is a tree, and $\frac{n(n-1)}{2}$ corresponds to the number of links in a complete graph. ■

Hence, using Proposition 3.1 and Corollary 3.2 we can show our main result.

Theorem 3.3: For the closed loop system (1)-(3) assume that $\mathcal{G}(t_0)$ is connected. Then, all agent velocities become asymptotically the same and collisions among agents are avoided.

Proof: By Corollary 3.2, the number of switching times of the closed loop system is finite and so the signal $\mathcal{G}(t)$ eventually becomes constant, i.e., $\mathcal{G}(t) \to \mathcal{G}$. It follows by Proposition 3.1 that if $\mathcal{G}(t_0)$ is connected, then $\mathcal{G}(t)$ is connected for all time $t \geq t_0$ and so eventually $\mathcal{G}(t) \to \mathcal{G} \in \mathcal{C}_n$. This observation implies that we can essentially study convergence of the system once the switching signal has converged and the network topology is fixed. As in Proposition 3.1, for any signal $\mathcal{G}$ the potential $V_{\mathcal{G}}$ is positive definite and,

$$\dot{V}_{\mathcal{G}} = -\mathbf{v}^T (L_{\mathcal{G}} \otimes I_m)\mathbf{v} = -\sum_{j=1}^m \mathbf{v}_{y_j}^T L_{\mathcal{G}} \mathbf{v}_{y_j} \leq 0$$

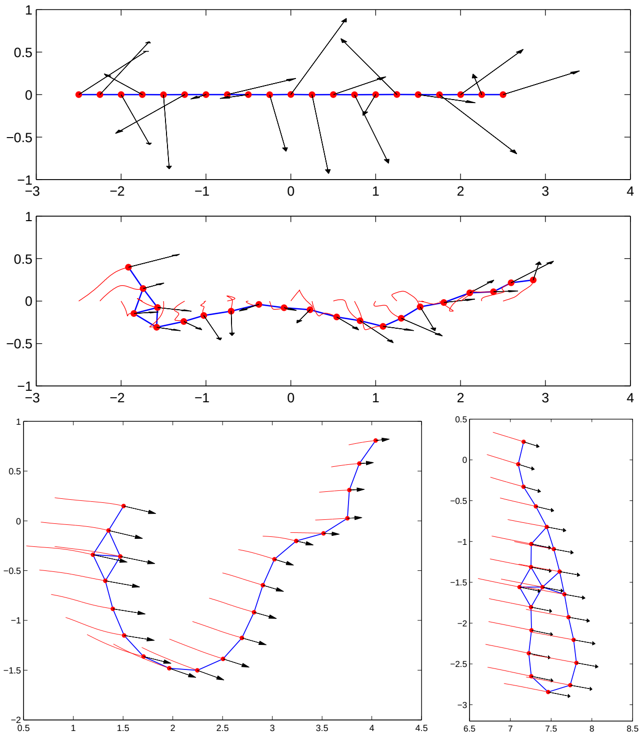


Fig. 3. Flocking of $n = 20$ agents.

since $\mathbf{v}_{y_j}^T L_{\mathcal{G}} \mathbf{v}_{y_j} \geq 0$ for all $j = 1, \ldots, m$, where $\mathbf{v}_{y_j} \in \mathbb{R}^n$ is the stack vector of the components of the agents' velocities along the y_j direction. Furthermore, the level sets of $\Omega_{\mathcal{G}}$ are closed by continuity of $V_{\mathcal{G}}$ in $D_{\mathcal{G}} \times \mathbb{R}_+^{mn}$. Note now that,

$$\begin{aligned} \Omega_{\mathcal{G}} &\subseteq \{\mathbf{v} \mid \|\mathbf{v}\|_2^2 \leq c\} \cap \left(\cap_{(i,j)\in\mathcal{E}} \{x_{ij} \mid V_{ij} \leq c\} \right) \\ &= \{\mathbf{v} \mid \|\mathbf{v}\|_2^2 \leq c\} \cap \left(\cap_{(i,j)\in\mathcal{E}} V_{ij}^{-1}([0, c]) \right) \triangleq \Omega \end{aligned}$$

The velocity set $\{\mathbf{v} \mid \|\mathbf{v}\|_2^2 \leq c\}$ is closed and bounded and hence, compact. Moreover, for all $(i, j) \in \mathcal{E}$ the sets $V_{ij}^{-1}([0, c])$ are closed by continuity of V_{ij} in the interval $(0, R)$. They are also bounded; to see this, suppose there exist indices i and j for which $V_{ij}^{-1}([0, c])$ is unbounded. Then, for any choice of $N \in (0, R)$, there exists an $x_{ij} \in V_{ij}^{-1}([0, c])$ such that $\|x_{ij}\|_2 > N$. Allowing $N \to R$, and given that $\lim_{\|x_{ij}\|_2 \to R} V_{ij} = \infty$, it follows that for any $M > 0$, there is a $N > 0$ such that $V_{ij} > M$. If we pick $M > c$ we reach a contradiction, since by definition $x_{ij} \in V_{ij}^{-1}([0, c]) = \{x_{ij} \mid V_{ij}(x_{ij}) \leq c\}$. Thus, all sets $V_{ij}^{-1}([0, c])$ are bounded and hence, compact. Therefore, Ω is compact as an intersection of finite compact sets. It follows that $\Omega_{\mathcal{G}}$ is also compact, as a closed subset of a compact set.

By LaSalle's invariance principle, every solution starting in $\Omega_{\mathcal{G}}$ asymptotically converges to the largest invariant set in $\{(\mathbf{x}, \mathbf{v}) \in D_{\mathcal{G}} \times \mathbb{R}_+^{mn} \mid \dot{V}_{\mathcal{G}} = 0\} = \{\mathbf{v} \in \mathbb{R}_+^{mn} \mid L_{\mathcal{G}} \mathbf{v}_{y_j} = \mathbf{0}, \ \forall j = 1, \ldots, m\}$. Since $\mathcal{G}$ is connected, the largest invariant set in $\{(\mathbf{x}, \mathbf{v}) \in D_{\mathcal{G}} \times \mathbb{R}_+^{mn} \mid \dot{V}_{\mathcal{G}} = 0\}$ is the set of velocity vectors $\mathbf{v} \in \mathbb{R}^{mn}$ such that $\mathbf{v}_{y_j} \in \text{span}\{\mathbf{1}\}$ for all $j = 1, \ldots, m$. In other words, the velocities of all agents in the switched system (1) asymptotically become the same.

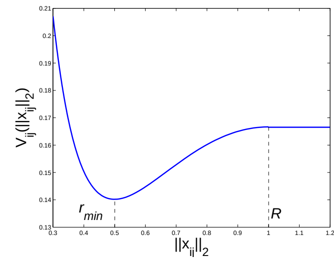


Fig. 4. Plot of the potential $V_{ij}(x_{ij})$ in [26] for $r_{min} = .5$ and $R = 1$.

Collision avoidance, on the other hand, is due to Proposition 3.1. ■

IV. Simulation Results

In this section we illustrate the proposed algorithm in various flocking scenarios where connectivity of the overall network can not be trivially maintained and show that the desired flocking motion of the agents is always achieved. Such scenarios may result from minimally connected initial configurations of the agents. In particular, we consider $n = 20$ agents in $\mathbb{R}^2$ initialized on a line with distances between adjacent agents equal to .25, initial velocities chosen randomly in the unit square, link range $R = .4$ and hysteresis $r = .3$ (Figure 3). Agents are denoted with dots, while links between the agents are indicated by solid lines. Moreover, the corresponding graphs do not explicitly depend on inter-agent distances, but correspond to the actual network topology according to Definition 2.2. Solid curves attached to every agent indicate the recently traveled paths, while arrows correspond to the agents' velocities.[6] Note that our approach guarantees connectivity of the network for all time and hence, asymptotic flocking of the group is achieved.

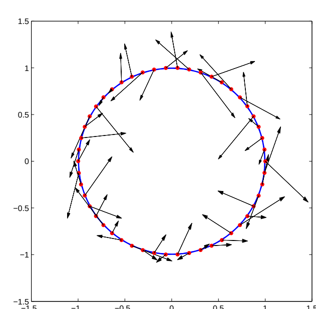
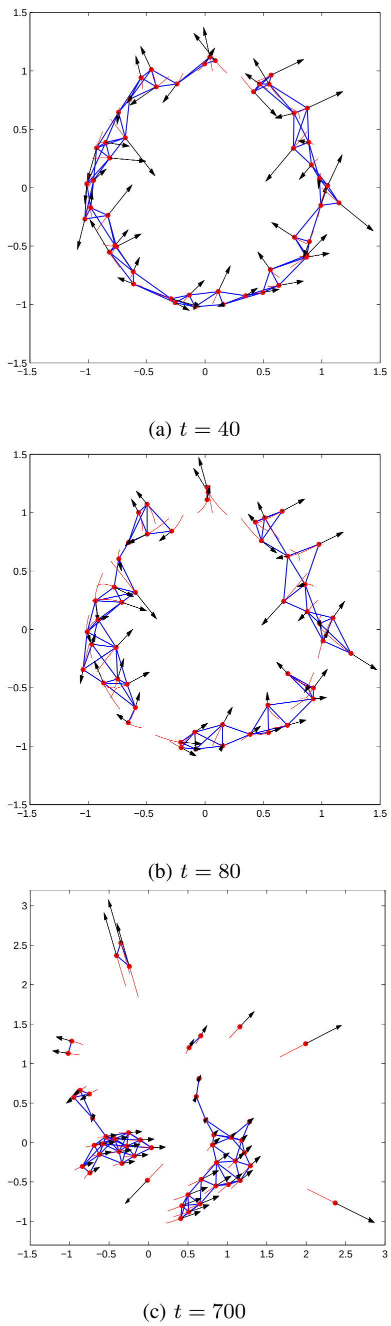


Fig. 5. 50 agents on the perimeter of a circle (Initial Configuration).

Our next scenario involves $n = 50$ agents in $\mathbb{R}^2$, symmetrically distributed on the perimeter of a circle of radius $d = 1.5$ having initial velocities chosen randomly in the unit square (Figure 5). The adjacency matrix of the corresponding

[6] Arrows are appropriately scaled for better illustration.



(a) $t = 40$

(b) $t = 80$

(c) $t = 700$

Fig. 6. Failure to flock without connectivity control.

network is,

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & \dots \\ 1 & 0 & 1 & 1 & 0 & 0 & \dots \\ 1 & 1 & 0 & 1 & 1 & 0 & \dots \\ 0 & 1 & 1 & 0 & 1 & 1 & \dots \\ 0 & 0 & 1 & 1 & 0 & 1 & \dots \\ 0 & 0 & 0 & 1 & 1 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

To best illustrate our approach, we compared it with previously suggested solutions that do not involve connectivity control of the network. In particular, we employed the approach followed in [26], where the main difference with respect to our framework lies in the choice of the potentials V_{ij}, which in [26] are defined as (Figure 4),

$$V_{ij}(x_{ij}) = \begin{cases} -\alpha_1\|x_{ij}\| + \log(\|x_{ij}\|) + \frac{\alpha_2}{\|x_{ij}\|}, & \|x_{ij}\| < R \\ -\alpha_1 R + \log(R) + \frac{\alpha_2}{R}, & \|x_{ij}\| \geq R \end{cases}$$

where $\alpha_1 = \frac{1}{r_{min}+R}$, $\alpha_2 = \frac{Rr_{min}}{r_{min}+R}$ and $0 < r_{min} < R$ is associated to the minimum value of V_{ij}. Figures 6 and 7 show the evolution of the system for the approach in [26] and our approach, respectively.

Note that in the absence of connectivity control the group of agents gets disconnected and fails to flock (Figures 6). On the other hand, our approach, guarantees connectivity of the network for all time and hence, asymptotic flocking of the group is achieved (Figures 7). Furthermore, no links are deleted from the network, while the final network topology contains 98 more links than the initial one, where $|\mathcal{E}(t_0)| = 200$. The link range and hysteresis used were $R = .4$ and $r = .3$, respectively. We finally mention scalability of our approach due to the large numbers of agents it can handle.

V. Conclusions

In this paper, we considered a distributed control framework to the multi-agent flocking problem that *simultaneously* addresses the desired velocity alignment as well as the connectivity requirement of the underlying network, necessary for alignment. The agents were modeled by dynamic point masses and the proposed control strategy involved potential fields able to achieve velocity alignment and ensure that switching in the model occurs among connected network topologies. The network topology was described by proximity graphs, modified to include a hysteresis in link additions, which enabled regulating the network topology. Under the assumption that the initial network is connected, the overall system was shown to asymptotically flock for all initial conditions, while collision avoidance was also guaranteed. Our approach was illustrated in various flocking scenarios and comparisons with other approaches from the literature demonstrated its robust nature. We believe that this work points to a new direction in distributed coordination of multi-agent systems, where robustness of solutions can be achieved through multi-objective control. Further research involves extending the proposed framework to more general switching schemes among connected network topologies that also account for link deletions.

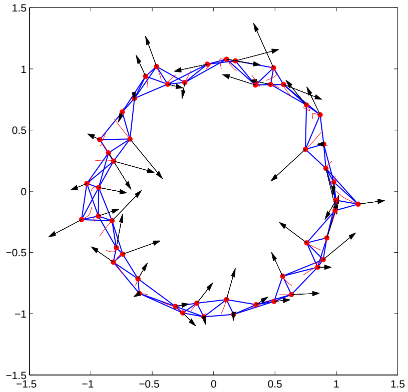
References

[1] J. K. Parrish and L. Edelstein-Keshet. *Complexity, Pattern, and Evolutionary Trade-Offs in Animal Aggregation*, Science, vol. 284, pp. 99-101, 1999.

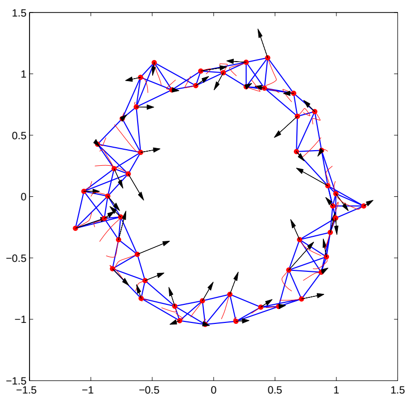
[2] C. Reynolds. *Flocks, Birds and Schools: A Distributed Behavioral Model*, Computer Graphics, vol. 21, pp. 25-34, 1987.

[3] T. Vicsek, A. Czirok, E. B. Jacob, I. Cohen and O. Schochet. *Novel Type of Phase Transitions in a System of Self-Driven Particles*, Physical Review Letters, vol. 75, pp. 1226-1229, 1995.

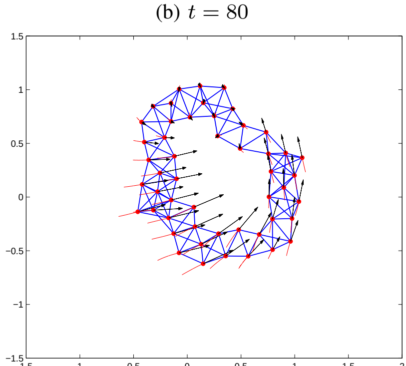
[4] J. P. Desai, J. P. Ostrowski and V. Kumar. *Modeling and Control of Formations of Nonholonomic Mobile Robots*, IEEE Transactions on Robotics and Automation, vol. 17(6), pp. 905-908, 2001.



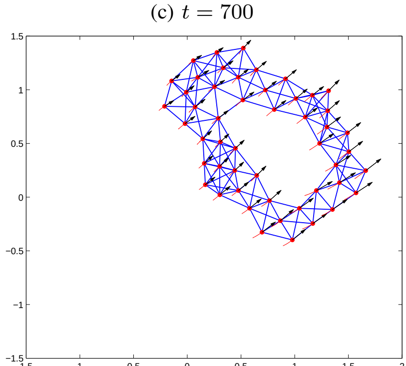
(a) $t = 40$



(b) $t = 80$



(c) $t = 700$



(d) $t = 2000$

Fig. 7. Successful flocking of 50 agents with connectivity control.

[5] P. Ogren, E. Fiorelli and N. E. Leonard. *Cooperative Control of Mobile Sensing Networks: Adaptive Gradient Climbing in a Distributed Environment*. IEEE Transactions on Automatic Control, vol. 49(8), pp. 1292-1302, August 2004.

[6] V. Gazi and K. M. Passino. *Stability Analysis of Swarms*, IEEE Transactions on Automatic Control, vol. 48(4), pp. 692-696, April 2003.

[7] P. Ogren, M. Egerstedt and X. Hu. *A Control Lyapunov Function Approach to Multi-Agent Coordination*, IEEE Transactions on Robotics and Automation, vol. 18(5), pp. 847-851, Oct. 2002.

[8] J. Cortes, S. Martinez and F. Bullo. *Robust Rendezvous for Mobile Autonomous Agents via Proximity Graphs in Arbitrary Dimensions*, IEEE Transactions on Automatic Control, vol. 51(8), pp. 1289-1298, 2006.

[9] L. Chaimowicz, N. Michael and V. Kumar. *Controlling Swarms of Robots Using Interpolated Implicit Functions*, Proceedings of the IEEE International Conference on Robotics and Automation, pp. 2498 - 2503, Barcelona, Spain, April 2005.

[10] R. Sepulchre, D. Paley, N. E. Leonard. *Stabilization of Planar Collective Motion: All-to-All Communication*, IEEE Transactions on Automatic Control, vol. 52(5), pp. 811-824, May 2007.

[11] G. Lafferriere, A. Williams, J. Caughman and J. J. P. Veerman. *Decentralized Control of Vehicle Formations*, Systems and Control Letters, vol. 54(9), pp. 899-910, Sept. 2005.

[12] T. Balch and R. C. Arkin. *Behavior-based Formation Control for Multirobot Teams*, IEEE Transactions on Robotics and Automation, vol. 14(6), pp. 926-939, Dec. 1998.

[13] W. Ren and R. Beard. *Consensus of Information under Dynamically changing Interaction Topologies*, Proceedings of the American Control Conference, pp. 4939-4944, June 2004.

[14] S. Poduri and G. S. Sukhatme. *Constrained Coverage for Mobile Sensor Networks*, In IEEE International Conference on Robotics and Automation, pp. 165-172, New Orleans, LA, May 2004.

[15] J. Lin, A.S. Morse and B.D.O. Anderson. *The Multi-Agent Rendezvous Problem*, Proceedings of the 42nd IEEE Conference on Decision and Control, pp. 1508-1513, Maui, Hawaii, Dec. 2003.

[16] D. V. Dimarogonas, S. G. Loizou, K. J. Kyriakopoulos and M. M. Zavlanos. *A Feedback Stabilization and Collision Avoidance Scheme for Multiple Independent Non-point Agents*, Automatica, vol. 42(2), pp. 229-243, Feb. 2006.

[17] D. P. Spanos and R. M. Murray. *Robust Connectivity of Networked Vehicles*, Proceedings of the 43rd IEEE Conference on Decision and Control, pp. 2893-2898, Bahamas, Dec. 2004.

[18] M. Ji and M. Egerstedt. *Distributed Formation Control while Preserving Connectedness*, Proceedings of the 45th IEEE Conference on Decision and Control, pp. 5962-5967, San Diego, CA, Dec. 2006.

[19] M. Mesbahi. *On State-dependent Dynamic Graphs and their Controllability Properties*, IEEE Transactions on Automatic Control, vol. 50(3), pp. 387-392, March 2005.

[20] M. C. DeGennaro and A. Jadbabaie. *Decentralized Control of Connectivity for Multi-Agent Systems*, Proceedings of the 45th IEEE Conference on Decision and Control, pp. 3628-3633, San Diego, CA, Dec. 2006.

[21] M. M. Zavlanos and G. J. Pappas. *Potential Fields for Maintaining Connectivity of Mobile Networks*, IEEE Transactions on Robotics, vol. 23(4), pp. 812-816, Aug. 2007.

[22] M. M. Zavlanos and G. J. Pappas. *Distributed Connectivity Control of Mobile Networks*, IEEE Conference on Desicion and Control, New Orleans, LA, Dec. 2007. To appear.

[23] R. Olfati-Saber and R. M. Murray. *Consensus Problems in Networks of Agents with Switching Topology and Time-Delays*. IEEE Transactions on Automatic Control, vol. 49(9), pp. 15201533, September 2004.

[24] A. Jadbabaie, J. Lin and A. S. Morse. *Coordination of Groups of Mobile Autonomous Agents using Nearest Neighbor Rules*, IEEE Transactions on Automatic Control, vol. 48(6), pp. 988-1001, 2003.

[25] N. Moshtagh, A. Jadbabaie, K. Daniilidis. *Vision-Based, Distributed Formation Control and Flocking of Multiagent Systems*, Robotics, Science and Systems, MIT, Cambridge, June 2005.

[26] H. Tanner, A Jadbabaie and G. Pappas. *Flocking in Fixed and Switching Networks*, IEEE Transactions on Automatic Control, vol. 52(5), pp. 863-868, May 2007.

[27] Norman Biggs, *Algebraic Graph Theory*, Cambridge University Press, 1993.